3-8 Finding and Estimating Square Roots

What You'll Learn

- To find square roots
- To estimate and use square roots

. . . And Why

To apply square roots in a real-world situation involving construction, as in Example 5

Check Skills You'll Need

GO for Help Lesson 1-2

Simplify each expression.

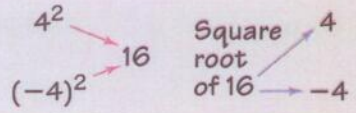
1. 11^2
2. $(-12)^2$
3. $-(12)^2$
4. 1.5^2
5. 0.6^2
6. $\left(\frac{1}{2}\right)^2$
7. $\left(-\frac{2}{3}\right)^2$
8. $\left(\frac{4}{5}\right)^2$

New Vocabulary

- square root
- principal square root
- negative square root
- radicand
- perfect squares

1 Finding Square Roots

The diagram at the right shows the relationship between squares and square roots. Every positive number has *two* square roots.



Key Concepts

Definition **Square Root**

The number a is a **square root** of b if $a^2 = b$.

Example $4^2 = 16$ and $(-4)^2 = 16$, so 4 and −4 are square roots of 16.

Vocabulary Tip

Read $\sqrt{16}$ as "the square root of 16." Read $\pm$ as "plus or minus."

A radical symbol $\sqrt{\ }$ indicates a square root. The expression $\sqrt{16}$ means the positive, or **principal square root** of 16. The expression $-\sqrt{16}$ means the **negative square root** of 16. The expression under the radical sign is a **radicand**. You can use the symbol $\pm$ to indicate both square roots.

1 EXAMPLE Simplifying Square Root Expressions

Simplify each expression.

a. $\sqrt{64} = 8$ — positive square root

b. $-\sqrt{100} = -10$ — negative square root

c. $\pm\sqrt{\frac{9}{16}} = \pm\frac{3}{4}$ — The square roots are $\frac{3}{4}$ and $-\frac{3}{4}$.

d. $\pm\sqrt{0} = 0$ — There is only one square root of 0.

e. $\sqrt{-16}$ is undefined. — For real numbers, the square root of a negative number is undefined.

Quick Check

1 Simplify each expression.

a. $\sqrt{49}$ b. $\pm\sqrt{36}$ c. $-\sqrt{121}$ d. $\sqrt{\frac{1}{25}}$

Vocabulary Tip

In decimal form, a rational number terminates or repeats. In decimal form, an irrational number continues without repeating.

Some square roots are rational numbers and some are irrational numbers.

Rational: $\sqrt{100} = 10$ $\quad \pm\sqrt{0.36} = \pm 0.6$ $\quad \sqrt{\frac{16}{121}} = \frac{4}{11}$

Irrational: $\sqrt{10} \approx 3.16227766$ $\quad \sqrt{\frac{1}{7}} \approx 0.377964473$

2 EXAMPLE Rational and Irrational Square Roots

Tell whether each expression is *rational* or *irrational*.

a. $\pm\sqrt{81} = \pm 9$ rational

b. $-\sqrt{1.44} = -1.2$ rational

c. $-\sqrt{5} \approx -2.23606798$ irrational

d. $\sqrt{\frac{4}{9}} = \frac{2}{3}$ rational

e. $\sqrt{\frac{1}{3}} \approx 0.57735026$ irrational

Quick Check 2 Tell whether each expression is *rational* or *irrational*.

a. $\sqrt{8}$ **b.** $\pm\sqrt{225}$ **c.** $-\sqrt{75}$ **d.** $\sqrt{\frac{1}{4}}$

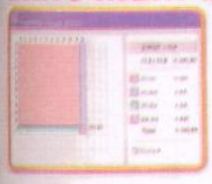
2 Estimating and Using Square Roots

The squares of integers are called **perfect squares.**

consecutive integers:	1	2	3	4	5	6
	↓	↓	↓	↓	↓	↓
consecutive perfect squares:	1	4	9	16	25	36

You can estimate square roots by using perfect squares.

Online active math



For: Square Root Activity
Use: Interactive Textbook, 3-8

3 EXAMPLE Estimating Square Roots

Estimation Between what two consecutive integers is $\sqrt{14.52}$?

$\sqrt{9} < \sqrt{14.52} < \sqrt{16}$ **14.52 is between the two consecutive perfect squares 9 and 16.**

$\downarrow \quad \downarrow \quad \downarrow$

$3 < \sqrt{14.52} < 4$ **The square roots of 9 and 16 are 3 and 4, respectively.**

$\sqrt{14.52}$ is between 3 and 4.

Quick Check 3 Between what two consecutive integers is $-\sqrt{105}$?

You can find the approximate value of a square root using a calculator.

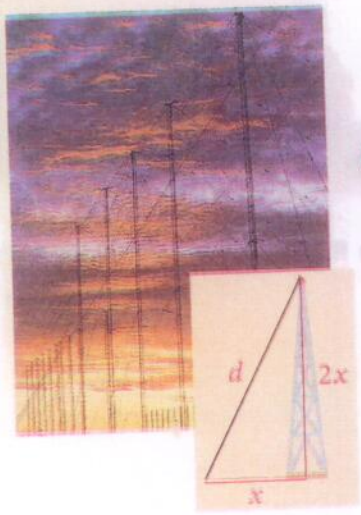
4 EXAMPLE Approximating Square Roots With a Calculator

Calculator Find $\sqrt{14.52}$ to the nearest hundredth.

$\sqrt{14.52} \approx 3.810511777$ **Use the calculator sequence** $\sqrt{\ }$ 14.52 ENTER.

≈ 3.81 **Round to the nearest hundredth.**

Quick Check 4 Find $\sqrt{17.81}$ to the nearest hundredth.



Many real-world formulas involve square roots.

5 EXAMPLE **Real-World Problem Solving**

Construction The formula $d = \sqrt{x^2 + (2x)^2}$ gives the length d of each wire for the tower at the left. Find the length of the wire if $x = 12$ ft.

$d = \sqrt{x^2 + (2x)^2}$

$d = \sqrt{12^2 + (2 \cdot 12)^2}$ Substitute 12 for x.

$d = \sqrt{144 + 576}$ Simplify.

$d = \sqrt{720}$

$d \approx 26.8$ Use a calculator. Round to the nearest tenth.

The wire is about 26.8 ft long.

Quick Check 5 Suppose the tower is 140 ft tall. How long is the supporting wire? Round to the nearest tenth of a foot.

EXERCISES

For more exercises, see *Extra Skill and Word Problem Practice*.

Practice and Problem Solving

A Practice by Example

Example 1 (page 176)

Example 2 (page 177)

Example 3 (page 177)

Example 4 (page 177)

Example 5 (page 178)

GO for Help

Simplify each expression.

1. $\sqrt{169}$
2. $\sqrt{400}$
3. $\sqrt{\frac{1}{9}}$
4. $\sqrt{900}$
5. $\sqrt{0.25}$
6. $\sqrt{\frac{36}{49}}$
7. $-\sqrt{1.21}$
8. $\sqrt{1.96}$
9. $\sqrt{0.36}$
10. $-\sqrt{144}$
11. $\sqrt{\frac{25}{16}}$
12. $\pm\sqrt{0.01}$

Tell whether each expression is *rational* or *irrational*.

13. $\sqrt{37}$
14. $-\sqrt{0.04}$
15. $\pm\sqrt{\frac{1}{5}}$
16. $-\sqrt{\frac{16}{121}}$

Between what two consecutive integers is each square root?

17. $\sqrt{35}$
18. $\sqrt{27}$
19. $-\sqrt{130}$
20. $\sqrt{170}$

Use a calculator to find each square root to the nearest hundredth.

21. $\sqrt{12}$
22. $-\sqrt{203}$
23. $\sqrt{11{,}550}$
24. $-\sqrt{150}$

25. **Sports** The elasticity coefficient e of a ball relates the height r of its rebound to the height h from which it is dropped. You can use the function $e = \sqrt{\frac{r}{h}}$ to find the elasticity coefficient. What is the elasticity coefficient of a tennis ball that rebounds 3 ft after it is dropped from a height of 3.5 ft? Round to the nearest hundredth.

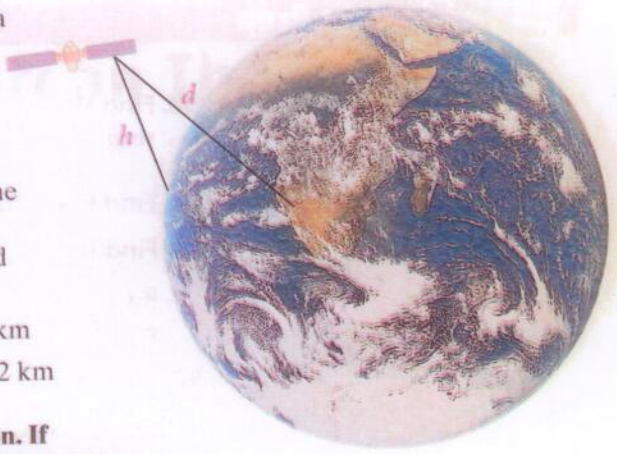
B Apply Your Skills

Find the square root(s) of each number.

26. 400
27. 0
28. 625
29. $\frac{9}{49}$
30. 1.69
31. $\frac{1}{81}$
32. 729
33. 2.25
34. 256
35. 0.01
36. $\frac{64}{121}$
37. 40,804

38. **Critical Thinking** What number other than 0 is its own square root?

39. Multiple Choice The formula $d = \sqrt{12{,}800h + h^2}$ gives the distance d in kilometers to the horizon from a satellite h kilometers above Earth. Find the distance to the horizon from a satellite 4200 km above Earth. Round to the nearest kilometer.



(A) 130 km (B) 7333 km

(C) 8450 km (D) 11,532 km

Find the value of each expression. If necessary, round to the nearest hundredth.

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40. $\sqrt{441}$ **41.** $-\sqrt{\frac{4}{25}}$ **42.** $\sqrt{2}$

43. $\sqrt{1.6}$ **44.** $-\sqrt{30}$ **45.** $-\sqrt{1089}$

46. $-\sqrt{0.64}$ **47.** $\sqrt{41}$ **48.** $\sqrt{75}$

49. Writing Explain the difference between $-\sqrt{1}$ and $\sqrt{1}$.

50. Open-Ended Find two integers a and b between 1 and 20 such that $a^2 + b^2$ is a perfect square.

51. Math in the Media In the cartoon, to what number is the golfer referring?

52. Physics If you drop an object, the time t in seconds that it takes to fall d feet is given by the formula $t = \sqrt{\frac{d}{16}}$.

a. Find the time it takes an object to fall 400 ft.

b. Find the time it takes an object to fall 1600 ft.

c. Critical Thinking In part (b), the object falls four times as far as in part (a). Does it take four times as long to fall? Explain.

Challenge

Critical Thinking **For Exercises 53–56, tell whether each statement is *true* or *false*. If the statement is false, give a counterexample.**

53. Every nonnegative number has two square roots.

54. The square root of a positive number is always less than the number.

55. The square root of an even perfect square is always an even number.

56. $\sqrt{p} + \sqrt{q} = \sqrt{p + q}$

57. a. What is the total area of the large square shown at the right?

b. What is the area of each shaded triangle?

c. What is the area of the shaded square?

d. What is the length of the diagonal of each 1×1 square?